INVERSE PROBLEMS OF HEAT EXCHANGE – FIELDS OF APPLICATION IN THE PLANNING AND TESTING OF TECHNICAL OBJECTS

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The article analyzes possibilities of the practical application of the method of inverse problems of heat exchange, a new trend of thermophysical investigations in the designing and testing of modern technical models.

Below we examine some general problems of thermophysical investigations in designing thermally loaded technical objects from the positions of the new methods of identifying* processes of heat exchange, in the first place methods based on the solution of inverse problems of heat exchange (IPHE) [1-3]. These methods find application in the most varied fields of science and technology: aviation and space flights, power engineering, nuclear engineering, chemical technology, machine building, etc. In view of the universal nature of the methods of identification under examination, we will chiefly touch upon the fields of designing and testing aircraft and spacecraft.

I. GENERAL CHARACTERISTIC OF THE METHODS AND FIELDS OF APPLICATION OF INVERSE PROBLEMS OF HEAT EXCHANGE

In devising new models of aircraft and space rockets, thermophysical investigations play an increasingly important role. This is due to the ever more stringent requirements concerning the effectiveness and reliability of systems of heat protection and heat regulation, reduction of the weight of the craft concerned, the need in designing and experimentally testing craft to take into account the ever more complex pattern of thermal interaction between the craft and the environment. This trend will most likely also be maintained in the future. This will, in particular, be due to the wider range of problems solved in space, and in connection with that the construction of reentry transport systems and launched craft [4].

The conditions of thermal interaction between the craft and the environment and the processes of internal heat exchange determine the selection of the type of the heat protection system, other systems of ensuring the required thermal regimes of the craft, the solution of problems of seeking rational design parameters of these systems, and in some cases also of more general problems of design concerning the configuration and principal design characteristics of the craft. The correctness of the solutions adopted in the course of the design process depends largely on the thoroughness and reliability of the investigation of heat-transfer phenomena, on the adequacy of the mathematical thermal models of real thermophysical processes occurring in the structure and heat protection of the craft. Especially important in this respect are investigations of heat regimes including bench tests and flight tests of heat-protective materials and structures. Such experiments and investigations are marked by great complexity, laboriousness, and high costs. It is then particularly important and essential to have available informative and reliable methods of thermophysical investigation, processing and analysis of experimental data. Among such methods are, in particular, the new methods based on the solution of the corresponding types of inverse problems of heat exchange.

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*The term "identification" has already "taken root" sufficiently both in system theory and in other fields of science and technology including thermal modeling and optimum control of heat-exchange processes [2].

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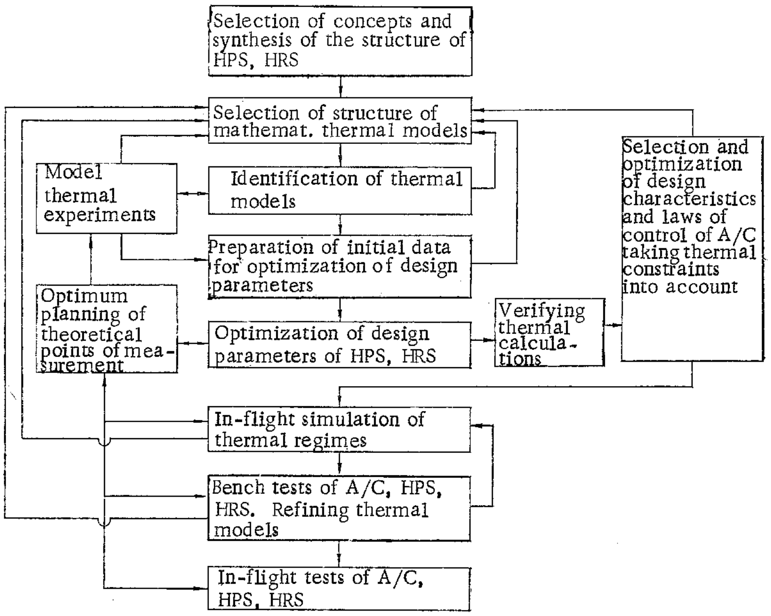

Fig. 1. Simplified block diagram of the process of thermal design and testing of the thermal regimes of aircraft: HPS) heat protection system; HRS) heat regulation system.

This does not mean that this class of methods supersedes other, more traditional approaches. We would like to draw attention to the presently spreading view that the introduction of methods of inverse problems altogether eliminates from practice as superfluous many "old" methods of experimentally investigating processes of heat exchange, e.g., some methods of determining heat fluxes and thermophysical characteristics. Such a view is erroneous. The methods of inverse problems of heat exchange were not devised in order to supersede existing and well-proven methods, but for use in those cases where traditional approaches were altogether inapplicable, or else did not yield sufficiently accurate results. In the last 20 or 30 years many such cases were brought to the fore by practice, in particular in the design and construction of new types of craft. A typical situation requiring the adoption of new methods was the modeling and investigation of highly intensive nonsteady-state regimes of heating craft, processes of heat and mass transfer in heat-protective materials and coatings. Under these conditions the existing methods of thermophysical measurement, processing and interpretation of experimental data were often useless because they had been devised for different conditions: usually for the investigation of steady-state or slowly changing processes of heat exchange with relatively low intensity.

The methods of inverse problems, worked out for new practical applications, also prove efficient in solving some traditional thermophysical and heat-engineering problems.

Thus, the new methods of investigating processes of heat exchange, based on the solution of inverse problems, appeared as an objective necessity. They broaden and complement the existing methodological apparatus of thermophysical investigations. To this we may add that in designing and testing thermally loaded structures, it is usual to have recourse to a whole complex of methods that complement and refine each other, and in this sense the trend under examination corresponds fully to actual fields of rational practical application.

Most widely used in practical investigations are now the inverse problems of heat conduction (IPHC), viz., boundary and coefficient problems, and also inverse problems of heat exchange in engineering systems.* Retrospective inverse problems have so far no wide practical application. Methods of solving and of practical application of geometric inverse problems of heat conduction, which apparently were formulated for the first time in [5, 2], are also only now beginning to be elaborated.

It should be pointed out that the planned and consistent development of methods of inverse problems, which at present is becoming a promising scientific and technical trend, is connected mainly with the investigation of thermal regimes in the design and experimental testing of craft. In this field in particular the main

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*We use the terminology of [2] whose author suggested a classification of kinds, statements, and methods of solving IPHE.

results were obtained and introduced in investigating the statement of IPHE and constructive methods of solving them. Many methods, initially worked out for the identification of thermal regimes of craft, later also found application in other branches of science and technology: in power engineering, metallurgy, chemical technology, etc.

Figure 1 shows the block diagram of thermal design and testing of the thermal regimes of craft showing the principal fields of application of the experimental methods including those based on IPHE. We will briefly deal with these fields.

Elaboration of the Mathematical Thermal Models of the Investigated Objects. Reliability of the mathematical models is one of the most important factors in the problem of adopting substantiated technical solutions in air- and spacecraft design, the devising of efficient methods and systems of automated design [6]. With increasing refinement of the configuration and design parameters of craft, the mathematical models are being continuously improved and broadened. Whereas at the initial design stage (in working out the technical propositions) it is often expedient to use the simplest models, it is necessary at the subsequent stages (draft project, technical project) to have recourse to more complete and accurate and as a rule also more complex models [7]. These models are also an instrument for carrying out verifying calculations of the thermal regime of the craft.

Thus, in designing a craft, some hierarchy of the mathematical thermal models (models of the processes of heat and mass exchange) is being examined, and the models of each level have to complement each other, have to be mutually coordinated. Questions of the elaboration of thermal models have to be given serious attention, and the modern apparatus of the system theory, the theory of graphs, etc. has to be used in the process.

By elaboration of a mathematical thermal model, we understand two successively solved problems: selection of the fundamental kind (structure) of the model and identification of its characteristics. It must be pointed out that the present state of the problem of the mathematical modeling of the thermal regimes of craft is characterized by the fact that in many cases it is possible to formulate the structure of the mathematical model which reflects accurately enough the real process of heat exchange [8-11]. However, such a structural model in connection with each actual case has a number of unknown characteristics (parameters or functions): boundary conditions, thermophysical properties, internal sources, etc. Experimental investigations are widely used to determine these characteristics of heat and mass exchange.

From the positions of the theory of optimum control, the problem of determining the unknown characteristics in the structural mathematical model according to some experimental information on the thermal regime of the object belongs to the problems of identification (inverse problems for dynamic systems). In applications to thermal investigations of craft, dynamic systems are used that are approximately described by ordinary differential equations and equations in partial derivatives, but the second kind of model (with distributed parameters) makes it possible in many cases to describe heat exchange processes more accutately.

A distinguishing feature of the thermal regimes of craft is their nonsteady state which at times is substantial. As was mentioned above, under these conditions the known steady-state or quasi-steady-state methods may yield unsatisfactory results in determining the characteristics of heat- and mass-exchange processes. A typical example of such a situation is connected with the analysis of the operation of heat protective coatings of composite materials during their process of destruction. A complicated complex of physicochemical transformations occurring in heat-protective materials is the reason that the characteristics of heat and mass exchange in the material depend on the heating rate [12]. The endeavor to determine these characteristics in the non-steady-state case makes it necessary to solve coefficient inverse problems for nonlinear systems describing the process of non-steady-state heat conduction in the heat-protective material, taking the physicochemical transformations into account. It is therefore a topical task to work out methods and algorithms for solving problems belonging to this class.

Methods of solving coefficient IPHC are at present being developed in two directions: by using analytical solutions of boundary problems of heat conduction and by constructing the corresponding numerical algorithms. In our opinion, the second direction is preferable in applications to thermal investigations of craft because it makes it possible to model a much wider range of practically important problems. Here, one of the promising approaches is the devising of the corresponding iteration procedures based on the gradient methods of minimization in combination with the parametrization of the sought functions [13].

Very important for practical applications is the complex problem of the identification of models describing the interaction of materials with a hot gas stream. From the data on measurements of the temperatures in the investigated material and also on the heat fluxes we have to estimate the characteristics of heat and mass exchange, both inside the material and on the surface interacting with the gas.

Preparation of the Initial Data for Designing Heat-Loaded Structures. We will show one of the characteristic problems of preparing the data whose solution may be based on the principles of IPHE. We are concerned now with the determination of thermal loads: the density of heat fluxes acting on different sections of the outer surface of a craft. In many cases, especially with the fairly complicated geometric shapes involved in spacecraft an accurate theoretical prediction of the thermal loads is usually difficult. The cause lies in the known "incalculability" of the conditions of external heat exchange connected with the transition of laminar flow into turbulent flow, the existence of zones of detachment of the flow, the formation of eddies, interference phenomena, a complex system of jumps in consolidation, etc. In such situations help is provided by experimental methods: models and dummies are subjected to special gasdynamic and other bench tests where the real conditions of heat exchange are being simulated. The necessity of modeling the non-steady-state process of heat exchange on the surface of the craft forces us to take recourse to non-steady-state methods of processing the experimental data. Here the traditional methods of measuring thermal loads fail in most cases, and new methods are needed that are based on the solution of inverse problems. In our case we can measure the temperature at points suitable for placing heat sensors, and by solving boundary IPHC, we can determine from the results of the measurements with a sufficient degree of accuracy the thermal loads (including non-steady-state ones) on the surfaces that are inaccessible for measurements.

Optimization of Design Parameters of Craft Taking Thermal Constraints into Account. Optimization of the Parameters of Heat-Protective Systems. The importance of this design stage is obvious: at this stage, in particular, are technical solutions being adopted within the framework of the selected concept of the system of heat protection, its design parameters are chosen, and some characteristics and regularities of spacecraft control are being refined. Such problems are distinguished by their great complexity and laboriousness, their close interrelation with problems of other design stages, in particular problems of selecting and refining the aerodynamic arrangement, investigation of the flight paths and algorithms of control of movement of the craft [14], analysis of the dynamic characteristics of the craft [15, 16], etc. For instance, the weight of the heat-protection system of the launched craft (and consequently of the entire craft) can be reduced not only by more rational design and more thorough testing of this system. The same effect can be attained by adequate selection of the aerodynamic shape and of the geometric parameters of the craft, and also as a result of reducing the thermal loads during launching in the atmosphere by selecting the required regularity of controlling the regime of motion of the craft.

At present one of the principal ways of improving the quality and efficiency of planning and design work is the automation of designing based on the use of computers [6]. The use of computers in seeking and adopting technical solutions makes it possible to analyze and select with more substantiation the parameters of the planned objects. An important trend of automated designing consists in working out methods of deliberate search for rational design solutions and parameters, proceeding from technical and technoeconomic criteria.

With the aid of high-speed computers on the basis of modern methods of computer mathematics, many problems concerning the planned structures under the conditions of highly intensive and non-steady-state thermal loads are being solved. An analysis of the thermal regimes of the structure obtained as a result of these calculations makes it possible to judge to a certain extent how rational the examined design variant is; however, it does not make it possible to ensure that the most rational variant will be found. That is due to the fact that similar calculations apply to the solution of direct problems, i.e., problems in which from specified (on the basis of the designer's intuition or of statistical data) thermophysical characteristics and geometric parameters of the structure, its thermal regimes are also calculated. In these problems there is no algorithm for the purposeful choice of design parameters in accordance with certain technical requirements.

Problems of optimizing the design parameters of thermally loaded structures are similar in statement to the extremal forms of inverse problems: the sought parameters are determined in accordance with some criterion represented in the form of a criterial function or functional, and also with a view to various kinds of constraining conditions. Naturally, such problems are more complex because they include the solution of the direct problem as well as an algorithm for the purposeful search for parameters of the structure.

In the mathematical formalization of such a problem, the planned structure may be described by a set of geometric parameters, thermophysical, mechanical properties, etc. The variety of these magnitudes forms a multidimensional space of parameters. Each design variant is determined by a set of the given magnitudes (this is usually some vector $\mathbf{p}$) and is characterized by some value of the selected criterion $F(\mathbf{p})$. In addition to that, any characteristic (e.g., temperatures T, stresses σ, etc.) or range of change of the sought parameters is subject to the corresponding constraints. Although by their physical nature these constraints usually differ, in the general case they may be described in the form of a system of inequalities $g_i(T, \mathbf{p}) \leq 0$, $i = \overline{1, n}$. Such inequalities define the region of admissible solutions because each of its points determines a design variant satisfying all the constraints. The optimum solution will be the one from the admissible region that has the smallest (largest) value of the criterion F(**p**).

In the general mathematical statement, the examined problems of selecting the optimum values of the design parameters correspond to problems of nonlinear programming for which by now many methods of solving them have been worked out. However, on account of the peculiarities of the mathematical models used in the design of thermally loaded structures, one of the chief problems of the practical realization of these methods is the devising of efficient procedures of seeking optimum solutions. Largely dependent on how successfully this problem is solved is the number of laborious (in terms of computer time) calculations of different variants of the designed structure. A possible way of improving the effectiveness of algorithms for automated thermal design consists in the application of methods of the theory of optimum control of systems with distributed parameters. We want to point out that some general approaches to the solution of problems of optimizing heat-protective structures may be based on methods worked out for solving IPHC, in particular on gradient algorithms of minimization with the procedure of calculating the gradient criterion of quality via the solution of the conjugate problem. A statement of the problem of optimum design of multilayered heat-protective coatings in the treatment similar to the inverse problem was given in [17], an algorithm for its solution using the specific mass of the coating as criterial function was presented in [18].

In-Flight Modeling of Thermal Regimes. Ground conditions of carrying out thermal experiments and tests are often unsuitable for reproducing satisfactorily the actual thermal regime of a spacecraft and its units taking into account the course of complex physicochemical processes behind the shock wave, in the boundary layer, and in the heat protection. Such a situation is typical of various kinds of launched craft. In similar situations it may prove useful to carry out flight tests with special models equipped with the required sensors and instruments [19, 20].

A research program of in-flight modeling of thermal regimes is usually considerably broader than the program of a thermal experiment carried out within the framework of flight design tests. The chief aims of in-flight modeling consist in verifying and assessing the accuracy of existing methods of calculating heat transfer used in the design of the heat protection, in determining the ranges of application of these methods, refining existing and devising new calculation methods, working out more efficient systems of heat protection. It may be assumed that the use of methods of IPHC will make it possible, with a fairly high degree of informativeness and the required degree of accuracy, to reconstruct the characteristics of the processes of heat exchange that interest us. In some cases such a way of obtaining essential information from thermal in-flight experiments is in fact the only way.

Bench Tests. Correction of Thermal Models. During their construction, spacecraft pass through a complex sequence of bench tests to which different components and units or even the craft as a whole are subjected. During the tests the surrounding flight conditions are modeled, the design and the systems of the craft are tested in order to improve their characteristics and reliability. This includes for many types of craft bench tests for investigating thermal regimes and verifying the effectiveness of operation of the systems of heat protection and heat regulation. Spacecraft in particular pass through laborious and complex thermovacuum tests.

During bench tests it becomes necessary to reconstruct fields of temperatures and heat fluxes in the tested object, to determine the intensity of internal heat sources and sinks and heat exchange coefficients between various elements of the craft. As initial experimental information, we usually have the known temperatures at a number of discrete points of the object. Thus, to obtain the required test results, it is indispensable to solve inverse problems of heat exchange in technical systems, and with their aid to refine (correct) the mathematical models of heat-exchange processes occurring in the sections and units of the craft [2, 10].

We want to point out that the necessity of solving similar problems of identification may not only arise in bench tests but also under conditions of space flight of a craft equipped with an adaptive heat control system. In this case the craft is also equipped with a board computer. Since the optical radiational characteristics of the outer surfaces of the craft and the coefficients characterizing the thermal connections between separate elements of the object change in time, the quantitative description of the model of the controlled heat exchange process is also bound to change. One way of improving the efficiency of operation of an automatic heat control system consists in correcting the coefficients of the thermal model introduced into the control algorithm. It is usually impossible to predict the changes of these coefficients, and these coefficients must therefore be estimated according to the data of temperature measurements carried out on board the craft. The corresponding inverse problems are solved by the board computer. Such an approach makes it possible to implement the adaptive method of optimum control of the thermal regime of the craft.

Flight Tests. During flight tests the construction of the craft passes through its final stages, and the craft's thermal regimes are subjected to final inspection. Here, the thermal state of the structure of the heat-protective coatings, etc., can be fairly reliably diagnosed with the aid of the methods of IPHE. In addition to that, the thermal state of individual elements of the structure and of the heat protection of the craft may also be checked under regular flight conditions or in flight path control according to the results of determining the characteristics of the external thermal regime (e.g., the surface temperature of the craft) obtained in real time from the solution of the boundary IPHC.

Thus, the principles and methods of IPHE are or may become an important means of thermophysical investigations at the most varied stages of the design and construction of modern spacecraft. The fullest and most effective use of the methods of inverse problems is possible in devising systems of automatic processing of data obtained in thermophysical investigations [21-24]. In this case the algorithms of solving IPHE are usually the basis of special mathematical provisions for ensuring the process of secondary processing of the experimental information.

II. PLANNING OF THERMOPHYSICAL EXPERIMENTS

Closely connected with the problem of efficient utilization of the methods of IPHE in experimental investigations is the problem of optimum planning of experiments. The theory and general methods of solving many problems of planning experiments have been fairly well worked out and are dealt with in the specialized literature. Less satisfactory is the situation as regards the practical application of these methods.

As regards the thermophysical investigations carried out during the construction of new technical models, of greatest interest is the planning of experiments designed to reveal the mechanism of phenomena. To this trend belong the experiments whose object is thermal modeling, i.e., composing and substantiating mathematical models of heat exchange processes. Here it should be pointed out that the use of the methods of planning is just as topical in physical as in purely mathematical thermal modeling.

In dependence on the kind of a priori information on a model we can formulate various experimental problems and the corresponding problems of the planning of experiments. For instance, if the structure of the thermal model is unknown, and the problem of recognition of this structure is being solved, then in its planning, the problem usually corresponds to discrimination of the model. If the kind of thermal model is specified, then the problem of determining the unknown characteristics of the model is solved. To this problem may correspond the planning of regression experiments.

An important stage in planning thermal experiments is the determination of the number and disposition of measurements in space and time (the problem of planning the measurements); the object of this stage is to obtain results with the required accuracy and with a minimum number of measurements. Here, in particular, is it of interest to work out methods using the apparatus of splines [25].

In planning thermal experiments, it is indispensable to take into account their technical peculiarities. It is therefore expedient to divide these experiments into two classes.

It is suggested to include in the first class those experiments which, on account of technicoeconomic considerations, do not permit the application of successive planning methods. These are thermal experiments carried out with complex and expensive models and actual objects or using unique bench-test equipment; all these experiments require large material expenditure.

As a rule, the possibilities of repeating such experiments are very limited. In these cases it is necessary to use static (a priori) methods of planning based on the fullest possible a priori information on the investigated phenomena and errors of measurement. Such information may be obtained in the process of numerical modeling of a physical phenomenon, and also from the results of analogous investigations and special model experiments.

The second class of experiments includes those which make it possible to carry out refining tests. In that case it is possible to use successive methods of planning, and the corresponding problems can be solved with fairly limited information. These are usually simple laboratory experiments connected with the testing of individual units or elements of structures, the investigation of the thermophysical characteristics of heat-protective materials, and the modeling of individual characteristics of heat exchange. To this class also belong experiments carried out in the process of regular operation or control of the object.

In addition to its own importance, the second class of experiments also plays a substantial role in preparing the information that is indispensable for planning experiments belonging to the first class.

We want to point out that devising methods of planning is also important from the point of view of constructing automated systems of controlling thermal tests.

III. THE STATE AND PROSPECTS OF FURTHER DEVELOPMENT OF METHODS OF SOLVING IPHE

One of the chief difficulties of solving inverse thermal problems and of the practical application of the corresponding methods consists in the fact that these problems in their initial "physical" formulation usually correspond to mathematically incorrectly stated problems. This situation is the result of infringement of causality in the transition from the statement of direct problems of heat exchange to the statement of inverse problems. The solution of direct problems in fact simulates real physical processes of heat transfer: on the basis of known causal characteristics the consequence is sought, whereas the statements of inverse problems, regardless of their absolutely concrete physical content, usually cannot be reproduced in the form of a real physical process. Especially important are therefore the analysis of the correctness of the statements of the IPHE, in particular revealing the mathematical conditions of the existence and uniqueness of the solution, and also the investigation of the possibility of implementing these conditions in experimental investigations. The third classical condition of correctness is the stability of the solution of the problems which requires that to infinitesimal changes of the input problems in the functional space of the input data correspond infinitesimal deviations of the solution in the solution space. A common characteristic feature of the input statements of various types of inverse problems of heat exchange is that the condition of stability of their solution is not fulfilled as a rule. Consequently, the methods of solving IPHE have to be regularized in some way [2, 26, 27].

By now the foundations have been laid of an applied theory and methodology of regularized solution of inverse problems of heat exchange, and a number of methods have been worked out for processing the experimental information and for identification of heat-transfer processes based on the regularization method of A. N. Tikhonov, methods of step-by-step and iteration regularization. Out of the fairly large number of newly devised approaches (see, e.g., the works [2, 28-34]), in our opinion the most interesting and promising is the class of experimental statements of inverse problems and of the corresponding methods of solving them. Our investigations showed that very universal and efficient are gradient algorithms of solving IPHE with regularizing stoppage of the iteration processes according to the condition of agreement between the number of approximations and the accuracy of specifying the initial information. The given method of iteration regularization of gradient algorithms was suggested in [35, 36] and substantiated in [37, 38], where it was established that for a fairly broad class of iteration methods there exist rules for ending iteration, with which they are regularizing algorithms after Tikhonov, and conditions were formulated of ending iteration processes for methods of the quickest descent, minimum discrepancies, and conjugate gradients. The new form of regularization of gradient algorithms suggested in [2, 39] yields approximations of the required order of smoothness.

The method of iteration regularization was made the basis for devising a complex of working methods and algorithms for solving boundary and coefficient inverse problems of heat conduction and inverse problems of heat exchange in engineering systems. These algorithms use effective methods of calculating the gradients of discrepancy functionals with the aid of solving the conjugate boundary problems. Methodological and experimental verification of the algorithms in question in connection with various practical situations showed that they are highly suitable both for linear and nonlinear IPHE, and that it is possible to take into account not only the qualitative but also the quantitative a priori information on the sought magnitudes.

In assessing the present state of the methods based on the solution of inverse heat-exchange problems, we may state that these methods are an effective means of thermophysical investigations, also in designing and testing thermally loaded engineering systems.

In regard to the requirements of practice in the plan of further development of this scientific trend, the following seems indispensable:

1) to continue the study and devising of methods of solving IPHC for cases of two-dimensional and three-dimensional heat propagation in solids;

2) to give much attention to the solution of non-steady-state boundary and coefficient IPHC for complex cases of heat transfer that include phenomena of heat and mass transfer both inside the body and on its surface;

3) to investigate the regions of possible practical application and methods of solving retrospective and geometric IPHC;

4) to continue the investigation and development of methods for solving inverse problems of heat exchange in engineering systems;

5) to carry out a qualitative and quantitative comparative analysis of different methods of solving inverse problems in order to discover the regions of the most rational practical application of these methods;

6) to carry out work in the further introduction of methods of inverse problems into experimental investigations and the identification of heat exchange processes in the designing and testing of thermally loaded technical objects;

7) to continue investigations and introduce the methods of optimum planning of thermophysical experiments and tests;

8) to pay particular attention to questions of the automation of information processing, using methods of inverse heat-exchange problems including the processing of experimental data in real time.

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SHOCK WAVES IN A LIQUID CONTAINING VAPOR BUBBLES

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The structure of shock waves in a liquid containing vapor bubbles is investigated, and an explanation is given for the mechanism of the anomalously high pressures in shock waves propagating in certain vapor–liquid media.

Condensation Shock

When a weak shock wave propagates in a liquid containing vapor bubbles, the temperature of the liquid does not change appreciably. Consequently, with the attendant increase in pressure and, hence, in the saturation temperature of the system the postshock vapor becomes supercooled, resulting in its condensation. Situations are therefore possible in which a shockwave reduces a two-phase mixture to a single-phase mixture.

Accordingly, the formulation of problems for bubbly liquids with allowance for the possibility of the annihilation of bubbles must incorporate sheets or boundaries $F^{(12)}$ separating regions of single- and two-phase flow. On these sheets $F^{(12)}$, which are aptly called condensation shocks, it is necessary to set up boundary conditions analogous to those on sheets of discontinuity.

We consider the stated conditions in a coordinate system wherein the sheet $F^{(12)}$ is at rest. The two-phase state (with bubbles) of the medium ahead of this shock is designated by the index 0, and the state of the medium

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